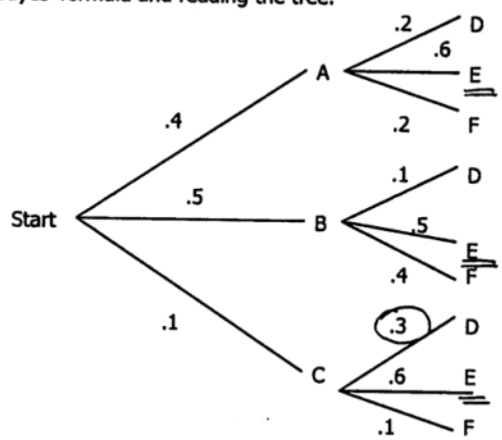
Thursday, December 08, 2011
2:25 PM

Solutions

Math 107 Fall 2011
TEST#3

Name: ____________________ **Signature:** ____________________

1. (20pts.) Find the probabilities in problems 1 – 4 by referring to the following tree diagram, using Bayes' formula and reading the tree.



1. $P(D|C)$ 2. $P(E)$ 3. $P(B|F)$

4. $P(B\cap E)$

(1) $P(D|C) = \boxed{0.3}$ (from the tree , directly)

(2) $P(E) = P(A\cap E) + P(B\cap E) + P(C\cap E) =$

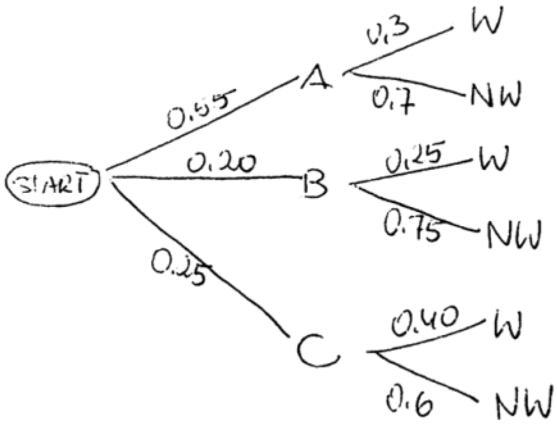
$$= (0.4)(0.6) + (0.5)(0.5) + (0.1)(0.6) = 0.24 + 0.25 + 0.06 = \boxed{0.55}$$

(3) $P(B|F) = \dfrac{P(F|B)\,P(B)}{P(F)} = \dfrac{(0.5)(0.4)}{(0.5)(0.4) + (0.4)(0.2) + (0.1)(0.1)} =$

$$= \frac{0.20}{0.20 + 0.08 + 0.01} = \boxed{\frac{20}{29}} = \boxed{0.6897}$$

(4) $P(B\cap E) = (0.5)(0.5) = \boxed{0.25}$

2.(20pts) An office supply company sells 3 different computer systems, A, B, and C. 55% of the customers who buy a computer buy brand A, 20% buy brand B, and 25% buy brand C. At the time of purchase the customer is offered an extended warranty for an added fee. 30% of the customers who bought brand A, 25% of the customers who bought brand B, and 40% of the customers who bought brand C bought the extended warranty. If a customer brings a computer system in for repair under the extended warranty, what is the probability that the computer was a brand A computer?(**draw the probability tree first**).



We want to find $\underline{\underline{P(A|W)}}$

$$P(A|W) = \frac{P(A)\,P(W|A)}{P(A)P(W|A) + P(B)\,P(W|B) + P(C)P(W|C)} =$$

$$= \frac{(0.55)(0.3)}{(0.55)(0.3) + (0.2)(0.25) + (0.25)(0.4)} = \frac{0.165}{0.165 + 0.05 + 0.1} =$$

$$= \frac{0.165}{0.315} = \boxed{0.5238}$$

3. (20pts.)An experiment consists of rolling two fair dice and adding the dots on the two sides facing up. Let X be the random variable equal to the sum of the dots. Assuming each simple event is as likely as any other,

a) find the probability distribution of X;
b) find the expected value, variance, and standard deviation of X.

Die#1 / Die#2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a)

X	2	3	4	5	6	7	8	9	10	11	12
P	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

b) $\underline{\underline{EX}} = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right) +$

$+ 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right) =$

$= \frac{1}{36}\left(2+6+12+20+30+42+40+36+30+22+12\right) =$

$= \frac{252}{36} = \boxed{7}$

$\sigma^2 = \underline{\underline{VarX}} = (2-7)^2\left(\frac{1}{36}\right) + (3-7)^2\left(\frac{2}{36}\right) + (4-7)^2\left(\frac{3}{36}\right) + (5-7)^2\left(\frac{4}{36}\right) + (6-7)^2\left(\frac{5}{36}\right) +$

$+ (7-7)^2\left(\frac{6}{36}\right) + (8-7)^2\left(\frac{5}{36}\right) + (9-7)^2\left(\frac{4}{36}\right) + (10-7)^2\left(\frac{3}{36}\right) + (11-7)^2\left(\frac{2}{36}\right) +$

$+ (12-7)^2\left(\frac{1}{36}\right) = \frac{1}{36}\left[25+32+27+16+5+0+5+16+27+32+25\right] =$

$= \frac{210}{36} = \boxed{5.8333}$

$\sigma = \sqrt{VarX} = \sqrt{5.8333} = \boxed{2.4152}$

4. (20) In a factory that assembles components for car radios the average time for an employee to assemble one component is 5 minutes with a standard deviation of 30 seconds. (Assume a normal distribution.) What is the percentage of employees who can assemble one component

a) in less than 4 minutes?

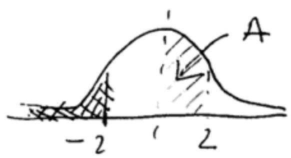
b) in more than 6 minutes?

c) between 4 and 6 minutes?

X – time to assemble components for car radios

$X \sim N(\underset{\text{mean}}{5},\ \underset{\text{standard deviation}}{0{,}5})$

a) $P(X<4) = P\left(\frac{X-5}{0{,}5} < \frac{4-5}{0{,}5}\right) = P(\underset{N(0,1)}{Z} < -2)$



$= P(Z>2) = \frac{1}{2} - P(A) = \frac{1}{2} - 0{,}4772 =$

$= \boxed{0{,}0228} \Rightarrow \boxed{2.28\ \%}$

b) $P(X>6) = P\left(\frac{X-5}{0{,}5} > \frac{6-5}{0{,}5}\right) = P(Z>2)$

$= \boxed{0{,}0228} \Rightarrow \boxed{2{,}28\ \%}$

c) $P(4<X<6) = 1 - P(X<4) - P(X>6) =$

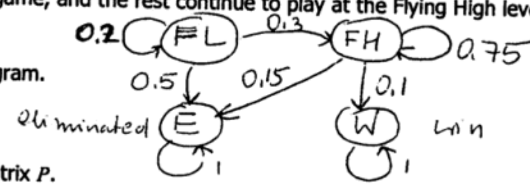
$= 1 - 0{,}0228 - 0{,}0228 = \boxed{0{,}9544} \Rightarrow \boxed{95{,}44\%}$

5. (20pts)ABSORBING MARKOV CHAINS

> The limiting matrix: $\overline{P} = \left[\begin{array}{c|c} I & 0 \\ \hline FR & 0 \end{array}\right]$
>
> where $F = (I - Q)^{-1}$ (F is called the fundamental matrix for P).

A computer game has two levels. Level one is called Flying Low and level two is called Flying High. To win the game the player must successfully complete Flying Low before getting to Flying High. On their first attempt at playing the game, 30% of players are able to successfully navigate Flying Low and move on to Flying High, 50% make an error and are eliminated from the game, and the rest continue to play at the Flying Low level. After making it to Flying High, 10% of the players successfully navigate Flying High and win the game, 15% make a fatal error and are eliminated from the game, and the rest continue to play at the Flying High level.

a) Draw a transition diagram.



b) Find the transition matrix P.

c) Write the transition matrix P in a standard form.

$$\begin{array}{c} \\ E \\ W \\ FL \\ FH \end{array} \begin{array}{c} \begin{array}{cccc} E & W & FL & FH \end{array} \\ \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0.5 & 0 & 0.2 & 0.3 \\ 0.15 & 0.1 & 0 & 0.75 \end{array}\right] \end{array}$$

d) Subdivide matrix P and then find matrix R and matrix Q.

$$R = \begin{bmatrix} 0.5 & 0 \\ 0.15 & 0.1 \end{bmatrix} \qquad Q = \begin{bmatrix} 0.2 & 0.3 \\ 0 & 0.75 \end{bmatrix}$$

e) Find matrix F. (Remember $F = (I - Q)^{-1}$).

$$I - Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.2 & 0.3 \\ 0 & 0.75 \end{bmatrix} = \begin{bmatrix} 0.8 & -0.3 \\ 0 & 0.25 \end{bmatrix}; \quad F = \frac{1}{0.2}\begin{bmatrix} 0.25 & 0.3 \\ 0 & 0.8 \end{bmatrix} = \begin{bmatrix} 1.25 & 1.5 \\ 0 & 4 \end{bmatrix}$$

f) Find FR.

$$FR = \begin{bmatrix} 1.25 & 1.5 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \\ 0.15 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.85 & 0.15 \\ 0.6 & 0.4 \end{bmatrix}$$

g) Write the limiting matrix $\overline{P}$.

$$\overline{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.85 & 0.15 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 \end{bmatrix}$$