Math 107 Fall 2011
TEST#2

Name: Solution **Signature:** ____________________

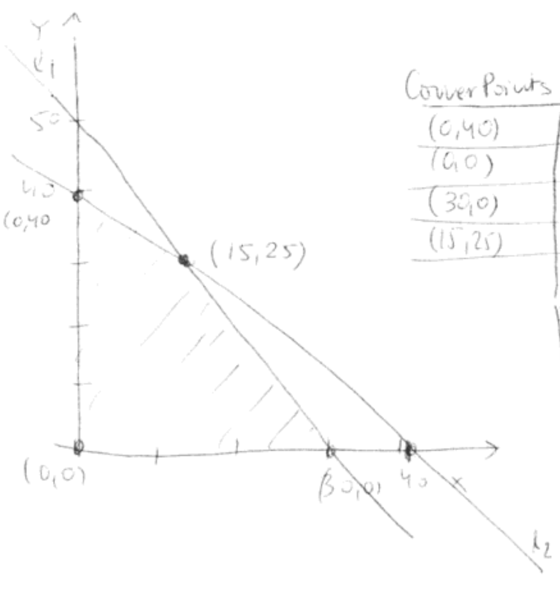
1.(25pts.) The top two dolls that a toy manufacturer makes are called Baby Wiggles and Sleepy Baby. To make a case of Baby Wiggles takes 5 units of raw material and 2 units of time to assemble. To make a case of Sleepy Baby takes 3 units of raw material and 2 units of time to assemble. On a given day the manufacturer has 150 units of raw material and 80 units of time. If the manufacturer makes a profit of $120 on each case of Baby Wiggles and $100 on each case of Sleepy Baby, how many cases of each type of doll should the manufacturer make in order to maximize profit.
Show your work. Set up the optimization problem first and then solve it.
You need to name your variables, state the objective function (which you maximize) and list all restrictions, i.e. inequalities which have to be fulfilled. Draw the feasible region and identify corner points. Then find the maximum profit and where it is obtained.

x = # of BW
y = # of SB

maximize $P = 120x + 100y$
subject to
$$\begin{cases} 5x + 3y \le 150 & \text{raw material} \\ 2x + 2y \le 80 & \text{assemble time} \\ x, y \ge 0 \end{cases}$$

l_1: $5x+3y=150$
$x=0 \Rightarrow y=50$
$y=0 \Rightarrow x=30$

l_2: $2x+2y=80$
$x=0 \Rightarrow y=40$
$y=0 \Rightarrow x=40$



Corner Points	$P=120x+100y$
(0,40)	4,000
(0,0)	0
(30,0)	3600
(15,25)	1800 + 2,500 = 4,300

l_1 & l_2: $\begin{cases} 5x+3y=150 \\ 2x+2y=80 \Rightarrow x = 40-y \end{cases}$
$5(40-y)+3y=150$
$200-5y+3y=150$
$50=2y \Rightarrow y=25$ & $x=15$

$\boxed{\text{The max profit is \$4,300 and is obtained when producing 15 of BW and 25 of SB}}$

2.(10pts.) In a lottery game where 5 numbers are chosen from the set {10, 11, 12, . . . , 28, 29} how many different 5-number cards could be where:

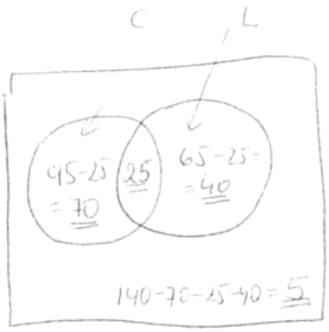
a) order is taken into consideration?
b) order is not taken into consideration?

a) We have 20 numbers, so

$$\# = P_{20,5} = \frac{20!}{15!} = 20 \times 19 \times 18 \times 17 \times 16 = 1,860,480$$

b) $$\# = \binom{20}{5} = \frac{20!}{15!\,5!} = \frac{1,860,480}{5 \cdot 4 \cdot 3 \cdot 2} = \frac{1,860,480}{120} = 15,504$$

3. (10p) A survey was done on a college campus to determine how many students owned a cell phone. Of the 140 students surveyed, 95 students owned a cell phone, 65 students had a land line, and 25 students had both a cell phone and a land line. How many students surveyed had neither a cell phone nor a land line? How many students had a cell phone but not a land line? (hint: a Venn diagram is helpful)



C - students with cell phones
L - - - - - - land lines

$n(C \cap L') = \underline{\underline{70}}$

$n(C' \cap L') = n((C \cup L)') = \underline{\underline{5}}$

4. (20pts.)An experiment consists of rolling two fair dice and adding the dots on the two sides facing up. Assuming each simple event is as likely as any other,

a) find the probability of the sum of the dots to be equal to 7;
b) find the probability of the sum of the dots to be not equal to7;

entries of the taple = sum of dots

1st roll / 2nd roll	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(\text{Sum} = 7) = \frac{6}{36} = \underline{\underline{\frac{1}{6}}}$$

$$P(\text{Sum} \neq 7) = 1 - \frac{1}{6} = \underline{\underline{\frac{5}{6}}}$$

5.(15p) From a standard deck of cards (52 cards) a 5-card hand is selected.
What is the probability of:

a) selecting 3 aces and two kings?
b) selecting at least 3 aces?

S = all 5-card hands

a) $P(\text{selecting 3 aces and 2 kings}) = \dfrac{\binom{4}{3} \times \binom{4}{2}}{\binom{52}{5}} =$

$$= \frac{4 \times \frac{\overset{4\cdot 3}{\cancel{4!}}}{2!\cancel{2}!}}{\frac{52!}{5!\,47!}} = \frac{2\cdot 4\cdot 3}{\frac{52\cdot 51\cdot \cancel{50}\cdot 49\cdot 48\,4}{\cancel{5\cdot 4\cdot 3\cdot 2}}} = \frac{24}{52\cdot 51\cdot 5\cdot 49\cdot 4} =$$

$$= \frac{24}{2{,}598{,}960} = \underline{0{,}00000923}$$

b) $P(\text{selecting at least 3 aces}) =$

$= P(\text{selecting 3 aces}) + P(\text{selecting 4 aces}) =$

$$= \frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} + \frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}} = \frac{4\cdot \frac{\overset{24}{\cancel{48}}\cdot 47}{\cancel{2}} + 48}{2{,}598{,}960} = \frac{4{,}560}{2{,}598{,}960} =$$

$$= \underline{\underline{0{,}001755}}$$

$10 \to$ 3 corrupted, 7 not corrupted

6. (20) A computer store has 10 copies of a software package. Unknown to the store, 3 of the packages have corrupted disks. What is the **probability** that

a. if a customer buys one package, that it will be one of the packages that contains a corrupted disk?

$$P = \frac{3}{10} = \underline{\underline{0.3}}$$

b. if a customer buys one package, that it will be one of the packages that is not corrupted?

$$P = \underline{\underline{\frac{7}{10}}}$$

c. if a customer buys three packages and all are corrupted.

$$P = \frac{\binom{3}{3}}{\binom{10}{3}} = \frac{1}{\frac{10!}{3!\,7!}} = \frac{1}{\frac{5\cdot 10\cdot 9\cdot 8}{2\cdot 3}} = \frac{1}{15\cdot 8} = \underline{\underline{\frac{1}{120}}}$$

d. if a customer buys three packages and two of them are corrupted.

$$P = \frac{\binom{3}{2}\binom{7}{1}}{\binom{10}{3}} = \frac{3\cdot 7}{120} = \frac{21}{120} = \underline{\underline{\frac{7}{40}}}$$

e. if a customer buys three packages and one of them is corrupted.

$$P = \frac{\binom{3}{1}\binom{7}{2}}{\binom{10}{3}} = \frac{3\cdot\frac{7\cdot 6}{2}}{120} = \frac{63}{120} = \underline{\underline{\frac{21}{40}}}$$

f. if a customer buys three packages and none of them are corrupted.

$$P = \frac{\binom{7}{3}}{\binom{10}{3}} = \frac{\frac{7!}{3!\,4!}}{120} = \frac{\frac{7\cdot 6\cdot 5}{2\cdot 3}}{120} = \frac{35}{120} = \underline{\underline{\frac{7}{24}}}$$

g. find the sum of the probabilities in c. – f. and write an explanation for the sum.

$$\Sigma = \frac{1}{120} + \frac{21}{120} + \frac{63}{120} + \frac{35}{120} = \frac{120}{120} = \underline{\underline{1}}$$

It has to be 1, since we sum over <u>all</u> possibilities