Math 107 Fall 2011
TEST#1

Name: Solutions **Signature:** ____________________

1. (15pts.) At $1.40 per pound, the daily supply of tobacco is 1,075 pounds and the daily demand is 580 pounds. When the price falls to $1.20 per pound, the daily supply decreases to 575 pounds and the daily demand increases to 980 pounds. Assume that the supply and demand equations are linear ($p = aq + b$) Find the supply equation.

$p = \$1.40 \Rightarrow q = 1{,}075$
$p = \$1.20 \Rightarrow q = 575$

$p = aq + b$, find a & b

$$\begin{cases} 1.40 = 1{,}075a + b \longrightarrow b = 1.40 - 1{,}075a \\ 1.20 = 575a + b \end{cases}$$

$1.20 = 575a + 1.40 - 1{,}075a$

$(1{,}075 - 575)a = 1.40 - 1.20$

$500a = 0.20$

$a = \frac{0.20}{500} = \underline{0.0004}$

$b = 1.40 - 1{,}075 \times 0.0004$

$\underline{b = 0.97}$

$\boxed{\text{S.E.: } p = 0.0004q + 0.97}$

2. (15pts.) Solve the following problems using Gauss-Jordan elimination.

$$\begin{cases} x_1 - x_2 + x_3 = 0 \\ 2x_1 + 3x_2 + 5x_3 = 1 \\ 3x_1 + 2x_2 + 4x_3 = 7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 3 & 5 & 1 \\ 3 & 2 & 4 & 7 \end{array}\right] \xrightarrow[-3R1+R3\to R3]{-2R1+R2\to R2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 5 & 3 & 1 \\ 0 & 5 & 1 & 7 \end{array}\right] \xrightarrow[\frac{1}{5}R3\to R3]{\frac{1}{5}R2\to R2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 3/5 & 1/5 \\ 0 & 1 & 1/5 & 7/5 \end{array}\right] \xrightarrow[-R2+R3\to R3]{R2+R1\to R1}$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 8/5 & 1/5 \\ 0 & 1 & 3/5 & 1/5 \\ 0 & 0 & -2/5 & 6/5 \end{array}\right] \xrightarrow{-5/2\, R3\to R3} \left[\begin{array}{ccc|c} 1 & 0 & 8/5 & 1/5 \\ 0 & 1 & 3/5 & 1/5 \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow[-8/5 R3+R1\to R1]{-3/5 R3+R2\to R2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$X_1 = 5$
$X_2 = 2$
$X_3 = -3$

3. (15pts.) Find the inverse of the following matrix. If the matrix does not have an inverse explain why.

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-3R1+R3 \to R3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 4 & -2 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow[-4R2+R3 \to R3]{R2+R1 \to R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & -10 & -4 & -3 & 1 \end{array}\right] \xrightarrow{-\frac{1}{10} R3 \to R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.4 & 0.3 & -0.1 \end{array}\right] \xrightarrow[-4R3+R1 \to R1]{-2R3+R2 \to R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -0.6 & -0.2 & 0.4 \\ 0 & 1 & 0 & 0.2 & -0.6 & 0.2 \\ 0 & 0 & 1 & 0.4 & 0.3 & -0.1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -0.6 & -0.2 & 0.4 \\ 0.2 & -0.6 & 0.2 \\ 0.4 & 0.3 & -0.1 \end{bmatrix}$$

check:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 2 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} -0.6 & -0.2 & 0.4 \\ 0.2 & -0.6 & 0.2 \\ 0.4 & 0.3 & -0.1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. (15pts.) Write the following system as a matrix equation and <u>solve by using inverses.</u>

$$\begin{cases} 3x_1 - 2x_2 = k_1 \\ x_1 - x_2 = k_2 \end{cases}$$

a) $k_1 = 12$, $k_2 = 5$

b) $k_1 = 6$ $k_2 = 4$

c) $k_1 = -8$ $k_2 = 3$

$$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}^{-1} = \frac{1}{-3+2} \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$$

a) $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 12 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

b) $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$$

c) $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -8 \\ 3 \end{bmatrix} = \begin{bmatrix} -14 \\ -17 \end{bmatrix}$$

5.(20pts) An economy is based on two industrial sectors, coal and steel. Production of a dollar's worth of coal requires an input of $0.40 from coal and $0.20 from steel. Production of a dollar's worth of steel requires an input of $0.40 from coal and $0.20 from steel. Find the output from each sector that is needed to satisfy a final demand of $25 billion for coal and $22 billion for steel.

a) Define the variables and find the variable matrix: X.
b) Find the technology matrix: M.
c) Find the final demand matrix: D.
d) Set up the matrix equation.
e) Solve the matrix equation in d).
f) State the answer to the question in sentence form.

x_1 = total production of C
x_2 = total production of S

$$M = \begin{array}{c c} & \begin{array}{cc} C & S \end{array} \\ \begin{array}{c} C \\ S \end{array} & \begin{bmatrix} 0.4 & 0.4 \\ 0.2 & 0.2 \end{bmatrix} \end{array} \qquad D = \begin{bmatrix} 25 \\ 22 \end{bmatrix} \text{ (in \$billions)}$$

$$X = MX + D$$
$$IX - MX = D$$
$$(I-M)X = D$$
$$\boxed{X = (I-M)^{-1} D}$$

$$I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.4 \\ 0.2 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.4 \\ -0.2 & 0.8 \end{bmatrix}$$

$$(I-M)^{-1} = \frac{1}{\underbrace{(.6)(.8) - (-0.4)(-0.2)}_{2.5}} \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0.5 & 1.5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0.5 & 1.5 \end{bmatrix} \begin{bmatrix} 25 \\ 22 \end{bmatrix} = \begin{bmatrix} 50 + 22 \\ 12.5 + 33 \end{bmatrix} = \begin{bmatrix} 72 \\ 45.5 \end{bmatrix}$$

x_1 = \$72 billions
x_2 = \$45.5 billions

For Problems 6-9, consider the following matrices

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

Problem 6. (5pts.) The dimension of the matrix A is :

(A) 3
(B) 3X3
(C) 6
(D) 9
(E) not defined

Problem 7.(5p.)Let the matrix F be defined as $F = C + D$. The (2,1) element of F, $f_{2,1}$ is:

(A) 1
(B) 2
(C) 3
(D) 4
(E) not defined

$0 + 1 = 1$

Problem 8. (5p.)Let the matrix G be defined as $G = A \, X \, B$. The (2,2)element of G, $g_{2,2}$ is:

(A) 1
(B) 2
(C) 3
(D) 4
(E) not defined

$$\begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \end{bmatrix}$$

Problem 9. (5pts.) Let the matrix H be the inverse of the matrix C. The (1,2)-element of H, $h_{1,2}$ is:

(A) -1
(B) 1
(C) 1/2
(D) -1/2
(E) not defined

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \frac{1}{1-0} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$